Quantum Theory Cannot Forbid Superluminal Signaling

Li YANG

State Key Laboratory of Information Security,

Graduate School of Chinese Academy of Sciences,

Beijing 100039,P. R. of China

April 7,2001

This note analyzes the angular distributions of the probabilities of two-photon states come out of the single-photon's stimulated emission amplification by means of a single-atom amplifier, to see that whether the quantum theory can forbid us exploiting EPR photon pairs combined with stimulated emission to realize superluminal signaling. Besides, we stressed that superluminal signaling will leads to a dilemma of causality in a system with two long superluminal channels and two short light channels.

It has been believed that the mathematical inseparability of the quantum theoretical representation is an essential part of nature, not a mere accident of the formalism[1]. However, the attempts to realize superluminal signaling over the last twenty years by means of EPR pairs has not been successful because one cannot clone a single particle in an unknown state sufficiently well [2]. Now let us see whether quantum theory can forbid superluminal signaling through a careful analysis of a concrete physical process.

Consider the stimulated emission of a single excited atom, or a single-atom light amplifier[3]. Suppose the angle between the polarization direction of an incoming single-photon flow and the atom's transition dipole moment $\overrightarrow{\mu}$ is θ , where $\overrightarrow{\mu}$ is perpendicular to the photon's wave vector. We note the photon's state by $|\theta\rangle$. After scattering, the system's two-photon state is of the form[3]

$$|\Psi_f^\theta\rangle = \alpha_\theta|2,0\rangle^\theta|g_\theta\rangle + \beta_\theta|1,1\rangle^\theta|g_{\theta+\frac{\pi}{2}}\rangle, \tag{1}$$

where $|2,0\rangle^\theta$ indicates that both photons are in the state $|\theta\rangle$, $|1,1\rangle^\theta$indicates a photon in each of the states $|\theta\rangle$ and $|\theta+\frac{\pi}{2}\rangle$, and $|g_\theta\rangle$ and $|g_{\theta+\frac{\pi}{2}}\rangle$ are the atom's final states. The state $|0,2\rangle^\theta$ would indicate that both photons are in the state $|\theta+\frac{\pi}{2}\rangle$. Let $|\Omega\rangle^\theta$ be a vector with three components $|2,0\rangle^\theta$,$|0,2\rangle^\theta$ and $|1,1\rangle^\theta$. We have $|\Omega\rangle^\theta = U(\theta)|\Omega\rangle^0$, where $U(\theta)$ is a unitary transformation,

$$|2,0\rangle^\theta = \cos^2\theta|2,0\rangle + \sin^2\theta|0,2\rangle + \frac{1}{\sqrt{2}}\sin 2\theta|1,1\rangle, \qquad (2a)$$

$$|0,2\rangle^\theta = \sin^2\theta|2,0\rangle + \cos^2\theta|0,2\rangle - \frac{1}{\sqrt{2}}\sin 2\theta|1,1\rangle, \qquad (2b)$$

$$|1,1\rangle^\theta = -\frac{1}{\sqrt{2}}\sin 2\theta|2,0\rangle + \frac{1}{\sqrt{2}}\sin 2\theta|0,2\rangle + \cos 2\theta|1,1\rangle. \qquad (2c)$$

Suppose $|g_\theta\rangle \neq |g_{\theta+\frac{\pi}{2}}\rangle$ and the incoming photons flow are a mixture of photons in $|\theta\rangle$ state and in $|\theta+\frac{\pi}{2}\rangle$ state with the same probability. The four transition probabilities is

$$w^\theta_{2,0} = 2\lambda^2\cos^2\theta d\Omega, \qquad (3a)$$

$$w^\theta_{1,1} = \lambda^2\sin^2\theta d\Omega, \qquad (3b)$$

$$w^{\theta+\frac{\pi}{2}}_{0,2} = 2\lambda^2\sin^2\theta d\Omega, \qquad (3c)$$

$$w^{\theta+\frac{\pi}{2}}_{1,1} = \lambda^2\cos^2\theta d\Omega. \qquad (3d)$$

where $w^\theta_{2,0}$ is the probability of generating a two-photon state $|2,0\rangle^\theta$ when incoming single-photon is in the state $|\theta\rangle$, $w^\theta_{1,1}$ is the probability of generating a two-photon state $|1,1\rangle^\theta$ when incoming single-photon is in the state $|\theta\rangle$, and so on; and

$$\lambda^2 = \frac{\omega^3\mu^2}{8\pi^2\hbar c^3}. \qquad (4)$$

Then by means of the unitary transformation $U(\theta)$ we can find the probability of the state $|2,0\rangle$ after scattering is

$$d\overline{\sigma^\theta_{2,0}} = \frac{1}{2}\lambda^2(1+\cos^2 2\theta)d\Omega. \qquad (5)$$

If we restrict our statistics to two-photon states, we have the probability of $|2,0\rangle$ state

$$\overline{P^\theta_{2,0}} = \frac{1}{3}(1+\cos^2 2\theta), \qquad (6a)$$

the other two probabilities are

$$\overline{P^{\theta}_{1,1}} = \frac{1}{3}, \qquad (6b)$$

$$\overline{P^{\theta}_{0,2}} = \frac{1}{3}\sin^2 2\theta. \qquad (6c)$$

[If let $|g_\theta\rangle = |g_{\theta+\frac{\pi}{2}}\rangle$, we have

$$\overline{P^{\theta}_{2,0}} = \frac{2}{3}\cos^2 2\theta, \qquad (7a)$$

$$\overline{P^{\theta}_{1,1}} = \frac{1}{3}, \qquad (7b)$$

$$\overline{P^{\theta}_{0,2}} = \frac{2}{3}\sin^2 2\theta. \qquad (7c)]$$

We can find from the analysis that one origin of the phenomenon that $\overline{P^{\theta}}$ is dependent on parameter θ is the zero point energy of the light field, and the other one is likely to be the Bose-Einstein statistics of photons.

The fact that $\overline{P^{\theta}}$ is dependent on the parameter θ means that with the single-atom light amplifier one can distinguish the parameter θ of an incoming single-photon flow by measuring the probabilities of $|2,0\rangle$ state or $|0,2\rangle$ state. If Alice and Bob share a sufficiently large number of two-photon EPR pairs in the Bell states with rotation invariance, they do not need another channel to complete their communication, then there is no law of physics which will obviously stop the superluminal signaling between Alice and Bob.

It can be shown that the superluminal signaling between long distance leads to a dilemma of causality in a system with two pairs of Alice and Bob belonged to two different inertial frames separately, and with four channels: two long superluminal channels [Alice(1),Bob(1)] and [Alice(2),Bob(2)],and two short light channels [Bob(1),Alice(2)]and [Bob(2),Alice[1]]. Then we know that if any phenomenon of superluminal signaling be found in experiment or theoretical analysis, it means directly something wrong with our understanding of nature.

Because of the success of Electrodynamics, General Relativity, and Quantum Field Theories we believe that the Lorentz covariance of our theories is right. Based on Lorentz covariance we can prove that any superluminal signaling will lead to the dilemma of causality. Then we believe that there is no superluminal signaling in nature. However, this conclusion does not imply that any non-relativistic theory cannot give out a result related with superluminal signaling at all. It is difficult to understand some people's belief that the non-relativistic quantum theory itself can forbid superluminal signaling automatically. One may argue that he believes there is no conflict between the theory of relativity and the theory of quantum mechanics, but many people(for example, J. A. Wheeler)

believe that the conflict is unavoidable. Considering the great difficulties we meet in the quantum field theory, especially in the quantum theory of gravity, we should realize that the conflict is evident and essential.

References

[1]Herbert,N. Found. Phys.12, 1171(1982). [2]Wootters, W. K. and Zurek W. H. Nature299, 802(1982). [3]Mandel, L. Nature304,188(1983).